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The dual of nothing

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We consider "bubbles of nothing" constructed by analytically continuing black hole solutions in anti–de Sitter space. These provide interesting examples of smooth time-dependent backgrounds which can be studied through the AdS/CFT correspondence. Our examples include bubbles constructed from Schwarzschild-AdS, Kerr-AdS and Reissner-Nordström AdS. The Schwarzschild bubble is dual to Yang-Mills theory on three-dimensional de Sitter space times a circle. We construct the boundary stress tensor of the bubble spacetime and relate it to the properties of field theory on de Sitter space.

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I. INTRODUCTION

There are many open questions in string theory, such as understanding cosmological evolution or the information flow in a black hole formation, for which the key element is a better understanding of dynamical spacetimes. There has recently been a surge of interest in studying string theory in time-dependent backgrounds. Several authors have discussed orbifold constructions giving solutions with tractable string theory descriptions [2]. These spacetimes contain singularities; this provides an opportunity to learn about novel singularity-resolution mechanisms in string theory, but it also makes these rather challenging examples. In another approach Sen has considered dynamical solutions of open string field theory with cosmological interpretations [3], but the corresponding spacetime solutions have not yet been understood. Against this context, it is useful to consider simpler spacetimes which exhibit interesting time dependence. In [1], Aharony, Fabinger, Horowitz and Silverstein pointed out that the double analytic continuation of Schwarzschild or Kerr spacetimes, dubbed "bubbles of nothing," provide interesting examples of smooth time-dependent solutions. Since these are vacuum solutions, they are consistent backgrounds for string theory at least to leading order.

It would also be interesting to find time-dependent asymptotically AdS solutions, as we could then use the AdS conformal field theory (CFT) correspondence to relate the time dependence to the behavior of the non-perturbative field theory dual. By relating this dynamical background to the dual field theory, it may be possible to sidestep, and get another perspective on, some of the difficult issues associated with studying string theory on these backgrounds, such as the possible appearance of non-local boundary interactions [1].

In this paper we will extend the work of [1] by considering the double analytic continuation of black hole solutions in AdS. These bubbles of nothing should then be related to some state in the field theory dual to string theory on $AdS_p \times S^q$. As we will see, this construction gives rise only to asymptotically locally AdS spacetimes and it would be interesting to find an example asymptotic to global AdS.

We will focus on the $AdS_5 \times S^5$ case, as this corresponds to the most well-understood field theory dual. For most of our results, there will be an obvious extension to the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ cases. It might seem that the $AdS_3 \times S^3$ case was equally interesting, but the double analytic continuation of the locally AdS_3 black hole solutions is just global AdS_3.[1]

We begin by studying the analytic continuation of time and an angle ($t \to i\chi, \theta \to i\tau$) of Schwarzschild-$AdS_5$ in Sec. II.[2] As in the flat space case, χ is periodically identified, and the resulting geometry is only asymptotically locally AdS (even though the proper length of the χ circle grows at large distance). We find that the natural conformal boundary of this spacetime is three-dimensional de Sitter space times a circle ($dS_3 \times S^1$). By the AdS/CFT correspondence, the Schwarzschild bubble should therefore be dual to $\mathcal{N}=4$ SU(N) Yang-Mills theory on $dS_3 \times S^1$. The characteristic exponential expansion of the bulk spacetime is therefore seen directly in the background for the field theory dual. We provide evidence for the duality by computing the boundary stress tensor of the bubble spacetime and relating it to the expectation value of the stress tensor of Yang-Mills theory in $dS_3 \times S^1$.

In Sec. III, we consider the extension to rotating black holes. Analytically continuing time, an angle and a rotation parameter ($t \to i\chi, \theta \to i\tau, a \to i\alpha$), we find that the presence of the negative cosmological constant introduces a qualitatively new feature compared to flat space: the metric has a coordinate singularity at a finite value of τ. It would be interesting to understand this breakdown of the metric in

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[1]This is related to the observation in [4] that the AdS soliton for $d=3$ is just global AdS_3.

[2]Related solutions were previously discussed in [4].

more detail. However, the extension to include rotation does not introduce any simplification: since the proper distance in all directions grows like r, the spacetime is still locally asymptotically AdS (unlike in flat space, where it was asymptotically flat), and the spacetime the dual field theory lives in still has an S^1 factor.

Finally, we consider the extension to charged black holes in AdS in Sec. IV. We think of this charge as arising from angular momentum on the S^5, so we consider the analytic continuation of time, an angle and the charge ($t \to i\chi, \theta \to i\tau, q \to i\varrho$), parallelling the discussion of Kerr-AdS. These charged cases are interesting because they have the same $dS_3 \times S^1$ conformal boundary, but there is now an additional parameter in the solution.

In Sec. V, we speculate about the interpretation of these results from the dual field theory point of view, and outline a program for future work. It is particularly appealing that the time dependence of the spacetime in these examples can be seen directly in the background for the dual field theory.

II. AdS-SCHWARZSCHILD BUBBLES

In this section we consider the bubbles obtained by analytic continuation of the AdS-Schwarzschild black hole. We will argue that these are related to SU(N) super YM (SYM) theory on a background which includes a de Sitter factor, and calculate the field theory stress tensor from the asymptotics of spacetime by the counterterm subtraction procedure. The 5D AdS-Schwarzschild black hole has a metric

$$ds^2 = -\left(1 + \frac{r^2}{l^2} - \frac{r_0^2}{r^2}\right)dt^2 + \left(1 + \frac{r^2}{l^2} - \frac{r_0^2}{r^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \cos^2\theta\, d\Omega_2^2), \tag{1}$$

where $d\Omega_2^2$ is the metric of the unit 2-sphere. We can analytically continue $t \to i\chi$ and $\theta \to i\tau$ to obtain another vacuum solution to gravity with a negative cosmological constant:[3]

$$ds^2 = \left(1 + \frac{r^2}{l^2} - \frac{r_0^2}{r^2}\right)d\chi^2 + \left(1 + \frac{r^2}{l^2} - \frac{r_0^2}{r^2}\right)^{-1} dr^2 + r^2(-d\tau^2 + \cosh^2\tau\, d\Omega_2^2). \tag{2}$$

To get a smooth spacetime, we require χ to be identified with period

$$\Delta\chi = \frac{2\pi l^2 r_+}{2r_+^2 + l^2}, \tag{3}$$

where r_+ is the minimum value of r,

$$r^2 \geq r_+^2 = \frac{l^2}{2}\left[-1 + \sqrt{1 + \frac{4r_0^2}{l^2}}\right]. \tag{4}$$

At any time τ, at fixed large r the space is the χ circle times a 2-sphere. As $r \to r_+$ the χ circle collapses, but the 2-sphere approaches a finite size $r_+^2 \cosh^2\tau$. This 2-sphere is the boundary of a bubble of nothing in AdS space which contracts from infinite size at $\tau = -\infty$ to a minimum size at $\tau = 0$ and then expands back out to infinite size as $\tau \to \infty$. The metric on the bubble boundary is that of 3d de Sitter space.

At large r, this metric will approach AdS locally. This is not obvious from the form of the asymptotic metric:

$$ds^2 \approx \left(1 + \frac{r^2}{l^2}\right)d\chi^2 + \left(1 + \frac{r^2}{l^2}\right)^{-1} dr^2 + r^2(-d\tau^2 + \cosh^2\tau\, d\Omega_2^2). \tag{5}$$

However, we can relate this to the usual embedding coordinates $X_1^2 + X_2^2 + X_3^3 + X_4^2 - T_1^2 - T_2^2 = -l^2$ by

$$\begin{aligned} X_2 &= r\cosh\tau\cos\theta\sin\phi, \\ X_3 &= r\cosh\tau\cos\theta\cos\phi, \\ X_4 &= r\cosh\tau\sin\theta, \\ T_2 &= r\sinh\tau, \\ X_1 &= (r^2 + l^2)^{1/2}\sinh\chi/l, \\ T_1 &= (r^2 + l^2)^{1/2}\cosh\chi/l. \end{aligned} \tag{6}$$

By contrast, the usual global AdS metric is

$$ds^2 = -\cosh^2\rho\, dt^2 + l^2 d\rho^2 + l^2\sinh^2\rho(d\psi^2 + \cos^2\psi\, d\Omega_2^2), \tag{7}$$

where $-\pi/2 < \psi < \pi/2$. This is related to the embedding coordinates by

$$\begin{aligned} X_2 &= l\sinh\rho\cos\psi\cos\theta\sin\phi, \\ X_3 &= l\sinh\rho\cos\psi\cos\theta\cos\phi, \\ X_4 &= l\sinh\rho\cos\psi\sin\theta, \\ X_1 &= l\sinh\rho\sin\psi, \\ T_2 &= l\cosh\rho\sin t/l, \\ T_1 &= l\cosh\rho\cos t/l. \end{aligned} \tag{8}$$

Thus, the time-dependent metric (5) is related to the standard global AdS coordinates (7) by

[3] In the string theory context, the solution of interest is the black hole $\times S^5$, with a constant Ramond-Ramond (RR) 5-form flux in both black hole and S^5 components. Since we analytically continue two coordinates in the black hole, the RR 5-form flux in this new spacetime will still be real.

$$r^2/l^2 = \sinh^2\rho \cos^2\psi - \cosh^2\rho \sin^2 t/l, \tag{9}$$

$$\sinh\tau = \frac{\cosh\rho \sin t/l}{[\sinh^2\rho \cos^2\psi - \cosh^2\rho \sin^2 t/l]^{1/2}}, \tag{10}$$

$$\sinh \chi/l = \frac{\sinh\rho \sin\psi}{[\sinh^2\rho \cos^2\psi - \cosh^2\rho \sin^2 t/l + 1]^{1/2}}. \tag{11}$$

To understand the asymptotic metric, consider (5) as a coordinatization of AdS, which we will call the time-dependent AdS coordinates. We see that these time-dependent coordinates do not even cover the entirety of a single period in global AdS: the coordinate patch has a boundary at $r=0$, corresponding to

$$\tanh\rho \cos\psi = \pm \sin t/l. \tag{12}$$

In particular, on the asymptotic boundary of the spacetime in global coordinates, $\rho \to \infty$, the boundary of the patch covered by the time-dependent coordinates is given by the null lines $\psi = \pm t/l \pm \pi/2$. We also see that in the time-dependent AdS coordinates, we should use the full range $-\infty < \chi < \infty$. As in the usual flat space case [6], the main effects of considering the exact metric (2) on the coordinates are twofold; the range of r is restricted to $r > r_+$ (which restricts us to a region of AdS covered by the time-dependent coordinates), and the spacetime is identified under $\chi \sim \chi + \Delta\chi$.

At large distances in AdS, i.e., as $\rho \to \infty$, the restriction to $r > r_+$ coincides with $\psi = \pm t/l \pm \pi/2$, the boundary of the time-dependent coordinate patch (up to exponential corrections in ρ). The action of the periodic identification of χ on the asymptotic metric is, however, slightly complicated. We will express it in terms of the global AdS coordinates. From Eq. (11), we can see that a surface $\chi = \chi_0$ in Eq. (5) corresponds to

$$\sin\psi = \frac{\tanh \chi_0/l}{\tanh\rho} \cos t/l. \tag{13}$$

In, for example, the $t=0$ slice, this surface will extend to the boundary along $\psi = \psi_0$ where $\sin\psi_0 = \tanh\chi_0/l$. It reaches a minimum value $\rho_{min} = \chi_0/l$ at $\psi = \pi/2$. Away from the region near the bubble, we can approximate the bubble solution (2) by the time-dependent AdS space (5), with these two restrictions. From the point of view of the usual AdS coordinates, the periodic identification in χ will identify two surfaces of the form (13), as depicted in Fig. 1. This looks pictorially rather like the construction of Bañados-Teitelboim-Zanelli (BTZ) from AdS_3, but identifying hypersurfaces rather than geodesics. Note, however, that this picture only takes into account the effects on the coordinates, and not the fact that the bubble geometry (2) differs from the time-dependent AdS metric (5) in the interior. If we just made these identifications on the time-dependent AdS metric (5), it would not be smooth at small r—in particular, in Fig. 1, it looks like there is a finite minimum distance between the two surfaces of fixed χ, but in the true bubble geometry (2), the distance between surfaces of fixed χ goes smoothly to zero.

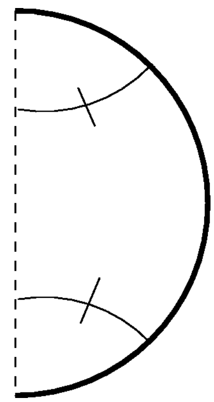

FIG. 1. Periodic identification of χ in global coordinates in the $t=0$ slice. The figure shows the radial coordinate in AdS and ψ. Over every point in the figure there is a 2-sphere. The locus of points of fixed χ is shown.

In the time-dependent AdS coordinates (5), the natural conformal compactification is a rescaling by l^2/r^2, giving a boundary metric

$$ds_\Sigma^2 = d\chi^2 + l^2(-d\tau^2 + \cosh^2\tau d\Omega_2^2). \tag{14}$$

This is a $(2+1)$-dimensional de Sitter space times S^1. Thus, if we assume the AdS/CFT correspondence can be extended to such asymptotically locally AdS cases, we should think of the dual description of this spacetime as given by some state of the SYM theory on $dS_3 \times S^1$. This can be related to the usual theory on $S^3 \times R$ obtained from global AdS by considering the boundary limit of the coordinate transformations (10), (11):

$$\sinh\tau = \frac{\sin t/l}{[\cos^2\psi - \sin^2 t/l]^{1/2}},$$
$$\sinh\chi/l = \frac{\sin\psi}{[\cos^2\psi - \sin^2 t/l]^{1/2}}. \tag{15}$$

These transformations take the metric (14) to

$$ds_\Sigma^2 = \frac{1}{(\cos^2\psi - \sin^2 t/l)} \times [-dt^2 + l^2(d\psi^2 + \cos^2\psi d\Omega_2^2)]. \tag{16}$$

Hence, from the boundary point of view, the coordinate transformation between time-dependent and global AdS coordinates involves a conformal rescaling by $\cos^2\psi - \sin^2 t/l$. This conformal factor vanishes at the boundary of the time-dependent AdS coordinate patch at $\psi = \pm t/l \pm \pi/2$, as expected. If we also consider the effect of the periodic identification in χ, by restricting to the fundamental region $-\Delta\chi/2 \le \chi \le \Delta\chi/2$, we find that this conformal factor is non-zero except at $t/l = \pm \pi/2$.

From the field theory point of view, there is a single dimensionless parameter: the ratio of the size of the S^1 to the radius of curvature of the de Sitter factor. This is just $\Delta\chi/l$, and to understand the physics from the field theory point of view, we should express all quantities in terms of this parameter. In fact, if we solve Eq. (3) for r_+ in terms of $\Delta\chi$, we find there are two roots:

$$r_+ = \frac{\pi l^2}{2\Delta\chi}\left[1 \pm \left(1 - \frac{2\Delta\chi^2}{\pi^2 l^2}\right)^{1/2}\right]. \tag{17}$$

In terms of the black hole solutions, this is just the usual statement that there is a minimum temperature for the black hole solutions, and there are two black holes for each temperature above that value—a smaller, unstable one and a larger stable one.

In the discussion of the flat space analogue in [1], it was argued that the bubble spacetime would be classically stable, but quantum mechanically unstable. Our expectations here are slightly different. For the larger root in Eq. (17), we would expect that the bubble will be both classically and quantum mechanically stable. The argument for classical stability is in the same spirit as [1]: the black hole solution is classically stable, so when one performs the analytic continuation, one expects to find no modes of the form $e^{ik\chi}$ with negative mass squared on the de Sitter factor (it would be useful, however, to check this explicitly).

The quantum instability in [1] was to the production of a widely separated bubble. First of all, we should note that the global AdS space (7) with two surfaces of the form (13) identified is not smooth. It is therefore not clear that we should give the bubble of nothing the same interpretation as a non-perturbative instability that the flat-space case had in [6]. Also, the presence of a negative cosmological constant implies widely separated objects cannot be treated independently. Finally, far from the original bubble, the χ direction has a large proper radius. As a result any identification of χ required to make a second bubble in the background of the first one will involve identifications over a very large proper length. This also suggests that there should be no instability to creating further bubbles.

For the smaller root in Eq. (17), on the other hand, there are signs of both classical and quantum instability. The corresponding black holes are thermodynamically unstable; it has been argued [7] that this corresponds to a dynamical instability. This may well lead to a classical instability of the bubble solution. Also, the solution with the larger root in Eq. (17) has lower energy, so we would expect the one with the smaller root to decay quantum mechanically into this larger bubble.

The dual field theory: Stress-energy tensor

We have shown that the asymptotic boundary of the bubble spacetime is $dS_3 \times S^1$. Therefore, by the AdS/CFT correspondence, we expect that $\mathcal{N}=4$ SU(N) Yang-Mills theory on $dS_3 \times S^1$ should be dual to the bubble. The time dependence of the bubble spacetime is reflected directly in the fact that the CFT lives on an expanding space. Here we will present evidence for this duality by comparing the CFT stress tensor to the boundary stress tensor calculated from the bulk spacetime using the counterterm subtraction procedure of [8,9].

Calculating the boundary stress tensor for the bubble spacetime (2) is a straightforward adaptation of the standard calculation of the boundary stress tensor for the Schwarzschild-AdS black hole. We must rescale the boundary stress tensor to express it in terms of the field theory in the boundary metric (14). The result for the bubble is

$$\begin{aligned} T^\chi_\chi &= -\frac{3}{16\pi G l^3}(r_0^2 + l^2/4) = -\frac{3N^2}{8\pi^2 l^4}\left(\frac{r_0^2}{l^2} + \frac{1}{4}\right),\\ T^\tau_\tau &= \frac{1}{16\pi G l^3}(r_0^2 + l^2/4) = \frac{N^2}{8\pi^2 l^4}\left(\frac{r_0^2}{l^2} + \frac{1}{4}\right),\\ T^\theta_\theta = T^\phi_\phi &= \frac{1}{16\pi G l^3}(r_0^2 + l^2/4) = \frac{N^2}{8\pi^2 l^4}\left(\frac{r_0^2}{l^2} + \frac{1}{4}\right), \end{aligned} \tag{18}$$

where in the second equality we have used the standard relation $l^3/G = 2N^2/\pi$ to rewrite the stress tensor in terms of field theory quantities. It is interesting to compare this to the corresponding result for the ordinary Schwarzschild-AdS case:

$$\begin{aligned} T^t_t &= -\frac{3}{16\pi G l^3}(r_0^2 + l^2/4) = -\frac{3N^2}{8\pi^2 l^4}\left(\frac{r_0^2}{l^2} + \frac{1}{4}\right),\\ T^\psi_\psi &= \frac{1}{16\pi G l^3}\, l(r_0^2 + l^2/4) = \frac{N^2}{8\pi^2 l^4}\left(\frac{r_0^2}{l^2} + \frac{1}{4}\right),\\ T^\theta_\theta = T^\phi_\phi &= \frac{1}{16\pi G l^3}(r_0^2 + l^2/4) = \frac{N^2}{8\pi^2 l^4}\left(\frac{r_0^2}{l^2} + \frac{1}{4}\right). \end{aligned} \tag{19}$$

The positive sign of the T^τ_τ component in Eq. (18) implies that this solution has a negative mass, while the negative T^χ_χ component is interpreted as a negative pressure (i.e., a tension) along this direction. The stress tensor is traceless, as in Schwarzschild-AdS. This is as expected, since the boundary metric is the product of a circle and a three-dimensional Einstein space, so the trace anomaly vanishes. Notice that the stress tensor has one piece that depends on the parameter r_0 and another that only depends on the cosmological constant. Below we will argue that the latter can be understood in the dual field theory as an anomaly contribution, while the former depends on the state.

Now, the dual description is in terms of $\mathcal{N}=4$ SYM on the $dS_3 \times S^1$ spacetime (14). This spacetime is conformally flat. We have already seen that the coordinate transformation (15) takes it to the form (16), which is conformal to the Einstein static universe; since flat space can be conformally embedded in the Einstein static universe, this implies that the boundary metric (14) is conformally flat. Since the spacetime is conformally flat, there is a standard result for the stress tensor [10]

$$\langle T^{\mu}_{\nu}\rangle = -\frac{1}{16\pi^2}(A^{(1)}H^{\mu}_{\nu} + B^{(3)}H^{\mu}_{\nu}) + \tilde{T}^{\mu}_{\nu}, \tag{20}$$

where ${}^{(1)}H^{\mu}_{\nu}$ and ${}^{(3)}H^{\mu}_{\nu}$ are conserved quantities constructed from the curvature (see [10] for their definitions), and $\tilde{T}^{\mu}_{\nu}$ is a traceless state-dependent part. For the $dS_3\times S^1$ space, the geometrical quantities are

$${}^{(1)}H^{\mu}_{\nu} = \frac{6}{l^4}\,\mathrm{diag}(-3,1,1,1) \tag{21}$$

and

$${}^{(3)}H^{\mu}_{\nu} = -\frac{1}{l^4}\,\mathrm{diag}(-3,1,1,1). \tag{22}$$

To fix the coefficients A and B, we compute the trace of Eq. (20),

$$\langle T^{\mu}_{\mu}\rangle = -\frac{1}{16\pi^2}[-6A\Box R - B(R_{\mu\nu}R^{\mu\nu} - 1/3R^2)] \tag{23}$$

and compare this to the conformal anomaly for $\mathcal{N}=4$ SYM [9,11]

$$\langle T^{\mu}_{\mu}\rangle = \frac{(N^2-1)}{64\pi^2}(2R_{\mu\nu}R^{\mu\nu} - 2/3R^2), \tag{24}$$

which fixes $A=0$ and $B=(N^2-1)/2$. As a result, the field theory stress tensor is

$$\langle T^{\mu}_{\nu}\rangle = \frac{(N^2-1)}{32\pi^2 l^4}\,\mathrm{diag}(-3,1,1,1) + \tilde{T}^{\mu}_{\nu}. \tag{25}$$

Thus the geometrical part of the stress tensor precisely reproduces the second term in Eq. (18) that is independent of the parameter r_0. This suggests that the state-dependent part of the field theory stress tensor should match the other term in Eq. (18), and should not produce an r_0-independent constant. It would be interesting to clearly identify the field theory states corresponding to the $r_0>0$ bubbles and show that this is the case.

Since we obtained the bubble spacetime by analytic continuation from a Euclidean solution, there is a natural vacuum state on the bulk spacetime defined by analytic continuation from the vacuum on the Euclidean spacetime. Similarly, there is a natural vacuum state in the field theory defined by analytic continuation from $S^3\times S^1$. It is presumably this Euclidean vacuum state we should be considering.[4]

We will defer detailed consideration of the field theory state to future work. Here, we will simply note that to compare to the field theory, we should rewrite the stress tensor in terms of the dimensionless parameter $\Delta\chi/l$. The form of the stress tensor rewritten in terms of $\Delta\chi/l$ is lengthy, so we will not give it explicitly; it can be easily obtained using Eqs. (4) and (17). Note that there are two roots in Eq. (17), giving two contributions to the partition function for a given $\Delta\chi/l$. We expect that the larger root will dominate for the field theory state obtained from the Euclidean vacuum.

[4]We are grateful to Djordje Minic for discussions on this subject.

III. KERR-AdS BUBBLES

The Schwarzschild-AdS bubble discussed above is asymptotically locally AdS; it would be interesting to identify asymptotically AdS solutions. In [1], the same issue for the Schwarzschild bubble was explored by adding rotation. Bubbles of nothing obtained by analytic continuation of Kerr spacetimes were also considered previously in [12]. We will now examine the effects of including a rotation parameter in the AdS case. We will find that, unlike the flat space case, this fails to remove the identification in the asymptotic region. There is also a new subtlety which arises from the presence of a negative cosmological constant.

To simplify comparison to the flat space treatment in [1], consider first the case $d=4$. Then the metric obtained by taking $t\to i\chi$, $a\to i\alpha$ and $\theta\to i\tau$ in the Kerr-AdS black hole [13] is

$$\begin{aligned} ds^2 = {} & \frac{\Delta_r}{\rho^2}\left[d\chi - \frac{\alpha}{(1+\alpha^2 l^{-2})}\cosh^2\tau\, d\phi\right]^2 \\ & + \frac{\rho^2}{\Delta_r}dr^2 - \frac{\rho^2}{\Delta_\tau}d\tau^2 + \cosh^2\tau\frac{\Delta_\tau}{\rho^2} \\ & \times\left[\alpha\, d\chi + \frac{(r^2-\alpha^2)}{(1+\alpha^2 l^{-2})}d\phi\right]^2 \end{aligned} \tag{26}$$

where

$$\rho^2 = r^2 + \alpha^2\sinh^2\tau, \tag{27}$$

$$\Delta_\tau = 1 - \frac{\alpha^2}{l^2}\sinh^2\tau \tag{28}$$

and

$$\Delta_r = (r^2-\alpha^2)\left(1+\frac{r^2}{l^2}\right) - 2Mr. \tag{29}$$

There is a bubble at $r=r_b$, where r_b is the largest root of Δ_r. But there is also now a breakdown of the metric at $\tau=\sinh^{-1}[l/\alpha]$, where Δ_τ vanishes. The curvature remains finite at this point, so it may be just a coordinate singularity. If we write $\sinh\tau = l/\alpha - \beta^2$, the leading-order β-dependent part of the metric is

$$\begin{aligned} & -\frac{2l\alpha(l^2+r^2)}{(l^2+\alpha^2)}d\beta^2 + \beta^2\frac{2(l^2+\alpha^2)}{l\alpha(l^2+r^2)} \\ & \times\left[\alpha\, d\chi + \frac{(r^2-\alpha^2)}{(1+\alpha^2 l^{-2})}d\phi\right]^2. \end{aligned} \tag{30}$$

For any given fixed value of r, this looks like the Rindler-like metric in the future light cone of a point. It therefore seems very likely that the singularity at $\beta=0$ corresponds to a horizon. However, a different combination of $d\chi$ and $d\phi$ is playing the role of the hyperbolic angle in the Rindler-like coordinates for each r, so it is difficult to find a coordinate transformation that takes us through the horizon.

Similar difficulties arise in the case $d=5$. The analytically-continued Kerr-AdS$_5$ solution gives the bubble metric

$$ds^2=\frac{\Delta_r}{\rho^2}\left[d\chi-\frac{\alpha}{(1+\alpha^2l^{-2})}\cosh^2\tau d\phi+\frac{\beta}{(1+\beta^2l^{-2})}\sinh^2\tau d\psi\right]^2+\frac{\rho^2}{\Delta_r}dr^2-\frac{\rho^2}{\Delta_\tau}d\tau^2+\cosh^2\tau\frac{\Delta_\tau}{\rho^2}\left[\alpha d\chi+\frac{(r^2-\alpha^2)}{(1+\alpha^2l^{-2})}d\phi\right]^2$$
$$-\sinh^2\tau\frac{\Delta_\tau}{\rho^2}\left[\beta d\chi+\frac{(r^2-\beta^2)}{(1+\beta^2l^{-2})}d\psi\right]^2-\frac{(1+r^2l^{-2})}{r^2\rho^2}\left[\alpha\beta d\chi+\frac{\beta(r^2-\alpha^2)\cosh^2\tau}{(1+\alpha^2l^{-2})}d\phi-\frac{\alpha(r^2-\beta^2)\sinh^2\tau}{(1+\beta^2l^{-2})}d\psi\right]^2 \quad (31)$$

where

$$\rho^2=r^2+\alpha^2\sinh^2\tau-\beta^2\cosh^2\tau, \quad (32)$$

$$\Delta_\tau=1-\frac{\alpha^2}{l^2}\sinh^2\tau+\frac{\beta^2}{l^2}\cosh^2\tau, \quad (33)$$

and

$$\Delta_r=\frac{1}{r^2}(r^2-\alpha^2)(r^2-\beta^2)\left(1+\frac{r^2}{l^2}\right)-r_0^2. \quad (34)$$

Here, the coordinates ϕ and ψ are angles with period 2π. If $\alpha>\beta$, there will be a breakdown of the metric where Δ_τ vanishes, as before. If $\beta>\alpha$, $\Delta_\tau>0$, but we now encounter problems where $\rho=0$.

There is still one case left, however: $\alpha=\beta$. This leads to a considerable simplification of the metric, which becomes

$$ds^2=\frac{\Delta_r}{\rho^2}\left[d\chi-\frac{\alpha}{(1+\alpha^2l^{-2})}(\cosh^2\tau d\phi-\sinh^2\tau d\psi)\right]^2+\frac{\rho^2}{\Delta_r}dr^2-\frac{\rho^2}{(1+\alpha^2l^{-2})}d\tau^2-\frac{\rho^2}{(1+\alpha^2l^{-2})}\cosh^2\tau\sinh^2\tau(d\phi-d\psi)^2$$
$$+\frac{1}{r^2}\left[\alpha d\chi+\frac{\rho^2}{(1+\alpha^2l^{-2})}(\cosh^2\tau d\phi-\sinh^2\tau d\psi)\right]^2, \quad (35)$$

where

$$\rho^2=r^2-\alpha^2 \quad (36)$$

and

$$\Delta_r=\frac{1}{r^2}(r^2-\alpha^2)^2\left(1+\frac{r^2}{l^2}\right)-r_0^2. \quad (37)$$

In this metric, we must restrict r to $r\geqslant r_+$, where r_+ is the largest root of

$$(r_+^2-\alpha^2)^2(r_+^2+l^2)=r_0^2l^2r_+^2. \quad (38)$$

(Note that this equation has roots for all real non-zero α, r_0, l.) The periodic identifications are

$$(\chi,\phi,\psi)\sim(\chi+\Delta\chi n_1,\phi+\Delta\chi\Omega n_1+2\pi n_2,\psi+\Delta\chi\Omega n_1+2\pi n_3), \quad (39)$$

where

$$\Omega=-\frac{\alpha(1+\alpha^2l^{-2})}{(r_+^2-\alpha^2)}. \quad (40)$$

The surface of the bubble is at $r=r_+$. The induced metric is

$$ds^2=-\frac{(r_+^2-\alpha^2)}{(1+\alpha^2l^{-2})}d\tau^2-\frac{(r_+^2-\alpha^2)}{(1+\alpha^2l^{-2})}\cosh^2\tau\sinh^2\tau(d\tilde{\phi}-d\tilde{\psi})^2+\frac{1}{r_+^2}\frac{(r_+^2-\alpha^2)^2}{(1+\alpha^2l^{-2})^2}(\cosh^2\tau d\tilde{\phi}-\sinh^2\tau d\tilde{\psi})^2, \quad (41)$$

where

$$\tilde{\phi}=\phi+\frac{\alpha(1+\alpha^2l^{-2})}{(r_+^2-\alpha^2)}\chi, \quad \tilde{\psi}=\psi+\frac{\alpha(1+\alpha^2l^{-2})}{(r_+^2-\alpha^2)}\chi. \tag{42}$$

It is also useful to consider a coordinate

$$\bar{\phi}=\tilde{\phi}-\tilde{\psi}. \tag{43}$$

In terms of $(\tau,\bar{\phi},\tilde{\psi})$ coordinates, the metric on the bubble is

$$\begin{aligned} ds^2=&-\frac{(r_+^2-\alpha^2)}{(1+\alpha^2l^{-2})}d\tau^2\\ &-\frac{(r_+^2-\alpha^2)}{(1+\alpha^2l^{-2})}\cosh^2\tau\sinh^2\tau d\bar{\phi}^2\\ &+\frac{1}{r_+^2}\frac{(r_+^2-\alpha^2)^2}{(1+\alpha^2l^{-2})^2}(\cosh^2\tau d\bar{\phi}+d\tilde{\psi})^2. \end{aligned} \tag{44}$$

We see that the constant τ slices of the bubble are tori. Unlike the non-rotating case, these tori are not all of finite size. The cycle parametrized by $\tilde{\psi}$ at fixed $\tilde{\phi}$ goes to zero size at $\tau=0$, as we can see from the first form of the metric. More worrisome, the $g_{\bar{\phi}\bar{\phi}}$ component in Eq. (44) is

$$\frac{\cosh^2\tau(r_+^2-\alpha^2)}{(1+\alpha^2l^{-2})^2}\left(\frac{(r_+^2-\alpha^2)}{r_+^2}\cosh^2\tau-(1+\alpha^2l^{-2})\sinh^2\tau\right), \tag{45}$$

so the cycle parametrized by $\bar{\phi}$ at fixed $\tilde{\psi}$ will go to zero size when τ satisfies

$$\tanh^2\tau=\frac{(r_+^2-\alpha^2)}{r^2(1+\alpha^2l^{-2})}, \tag{46}$$

and becomes timelike for larger values of τ. We will leave the resolution of these difficulties for future work.[5]

As a general comment, we note that even if we had better examples, adding rotation would not remove the asymptotic identification. In the flat space case, proper lengths in the χ direction are asymptotically constant, while proper lengths in the ϕ,ψ directions grow linearly in r at large distances. Thus, the circle in the χ direction formed by the identification (39) would have divergent size at large r for non-zero Ω. In the anti–de Sitter case, however, proper lengths in the χ and sphere directions all grow linearly in r, but this growth is removed by the conformal rescaling to obtain a boundary metric. Hence, replacing the identification (3) by (39) will not eliminate identifications in the conformal boundary; the Kerr-AdS bubble spacetimes are still only asymptotically locally AdS.

[5]See, however, [5] for a construction in higher dimensions.

IV. REISSNER-NORDSTRÖM AdS BUBBLES

Since we are interested in considering bubbles in the context of the AdS/CFT correspondence, and hence in spacetimes which are asymptotically $AdS_5\times S^5$, there is another possibility to consider: we can add angular momentum on the S^5. From the five-dimensional point of view, this corresponds to considering charged black holes: a particularly simple example is to add three equal commuting angular momenta, which will give electrically charged Reissner-Nordström AdS black holes [14]. This leads to new examples with the same asymptotic structure as in the Schwarzschild-AdS case.

Performing the analytic continuations $t\to i\chi$, $\theta\to i\tau$, $q\to i\varrho$ on the solution of [14] gives us the bubble solution[6]

$$\begin{aligned} ds^2=&\left(1+\frac{r^2}{l^2}-\frac{r_0^2}{r^2}-\frac{\varrho^2}{r^4}\right)d\chi^2\\ &+\left(1+\frac{r^2}{l^2}-\frac{r_0^2}{r^2}-\frac{\varrho^2}{r^4}\right)^{-1}dr^2\\ &+r^2(-d\tau^2+\cosh^2\tau d\Omega_2^2) \end{aligned} \tag{47}$$

with the gauge field

$$A_\chi=\frac{\sqrt{3}\varrho}{2r^2}-\frac{\sqrt{3}\varrho}{2r_+^2}. \tag{48}$$

As in the Schwarzschild case, we need to periodically identify χ with period

$$\Delta\chi=\frac{2\pi l^2r_+^5}{2r_+^6+r_+^4l^2+2\varrho^2l^2}, \tag{49}$$

where r_+ is the largest root of

$$\frac{r_+^6}{l^2}+r_+^4-r_0^2r_+^2-\varrho^2=0. \tag{50}$$

Note that this equation has a solution for r_+ for all r_0 and ϱ; as in flat space examples, the analytic continuation of q eliminates the possibility that there is no root.

The effects of ϱ in the metric are negligible at large r, so the asymptotic structure of this spacetime is the same as the uncharged case, and we get the same $dS_3\times S^1$ metric (14) on the conformal boundary. Here, we can think of $\Delta\chi/l$ and ϱ (which is an R charge in the CFT) as the appropriate parameters.

We can determine the branch structure by considering the behavior of $\Delta\chi$ as a function of r_+. For small and large r_+,

[6]We must analytically continue the charge so that the resulting ten-dimensional metric is real.

$\Delta\chi \to 0$. There will be a maximum where $\Delta\chi' = 0$, which gives

$$2r_+^6 - r_+^4 l^2 - 10\varrho^2 l^2 = 0. \tag{51}$$

Since this equation has only one real root, $\Delta\chi(r_+)$ has a single maximum. Below this maximum value, there are two solutions for r_+ for given $\Delta\chi$, as in the uncharged case. (Note that this branch structure is quite different from that obtained for real q.) It would be interesting to explore the stability of these solutions as well.

Since the boundary stress tensor is independent of subleading terms in the metric, it will have the same form as in the uncharged case (18). However, because Eq. (49) gives us a sixth-order polynomial to solve for r_+, we cannot write the stress tensor explicitly in terms of $\Delta\chi$ and ϱ.

V. CONCLUSIONS

We have begun an investigation of time-dependent bulk spacetimes in the context of the AdS/CFT correspondence. Inspired by the work of [1], we have considered the smooth bubble solutions obtained from analytic continuation of bulk black hole solutions. This gives asymptotically locally AdS spacetimes which are dual to field theory on simple time-dependent backgrounds. The fact that the time dependence of the bulk spacetime shows up as time dependence in the background for the dual field theory is very encouraging. It suggests that there may be interesting connections between, for example, the notions of particle creation on the two sides of the duality.

We focused on asymptotically $\mathrm{AdS}_5 \times \mathrm{S}^5$ spacetimes, dual to $\mathcal{N}=4$ SYM theory. The extension to other cases of interest should be straightforward. For the simplest example, the Schwarzschild-AdS_5 bubble, the dual field theory lives on three-dimensional de Sitter space cross a circle. We calculated the boundary stress tensor of the bubble spacetime and showed that that it had two pieces, one which depended on the parameters of the bubble, and the other which was universal. We showed that this universal part is reproduced by the universal anomaly contribution to the stress tensor of Yang-Mills theory on $\mathrm{dS}_3 \times \mathrm{S}^1$. It will be very interesting for the future to understand the bubble parameter dependent part of the boundary stress tensor from the dual perspective. For a given radius of the circle, there are two bubble solutions, with different values for the minimum radius of the bubble. We expect that the smaller bubble solution should be unstable. A careful analysis of the classical perturbations of both these solutions, along the lines of the analysis of the AdS solitons in [4], would be very useful.

We extended the construction to include angular momentum both in the AdS factor and on the S^5. For angular momentum on the AdS factor, there are new coordinate singularities which appear after analytic continuation for generic values of the parameters. These seem to be associated with a horizon in the bulk spacetime, but we did not attempt to resolve this issue in detail. Angular momentum on the S^5 is more tractable, and leads to a structure which is very similar to the Schwarzschild-AdS case, but with an additional parameter. Varying this parameter provides additional opportunities to study the behavior of the dual field theory. We note that unlike in [1], adding either kind of angular momentum does not simplify the asymptotic structure of the spacetime or alter the late-time behavior of the bubble.

The main direction for future work is to study the properties of the field theory on $\mathrm{dS}_3 \times \mathrm{S}^1$, and attempt to relate the vacuum states on that background to the bulk spacetimes discussed here. It would be very interesting if particle production in the bulk and on the boundary could be related. Another open area is to attempt to find constructions that give tractable time-dependent asymptotically AdS solutions, which could be related to the field theory on R^4.

It would also be interesting to consider the analogues of the construction in [2], quotienting AdS by a timelike or a null isometry. However, since these isometries also act on the conformal boundary, this will lead to identifications in the dual as well, and it may be difficult to deal with the resulting backgrounds in the field theory.

Note added. While this paper was in preparation, Ref. [5] appeared, which discusses some of the same solutions.

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